Calculating moist enthalpy revisited

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Abstract :

In a guest post [ref.2] published in July a single line formula was constructed to compute the moist enthalpy of surface air from the usual meteorological measurements of dry temperature, air pressure and relative humidity. Prof. Fucaloro showed in a following post [ref.3] that starting from a table of saturated water vapour pressure, an expression for the latent heat could be derived from the basic Clausius-Clapeyron equation. The question was asked if the linear expression given for the latent heat L in function of temperature was sufficiently precise, and if using a less coarse stepped water vapour table as that in the first post would not give a better result. These suggestions are followed in this post, leading to a quadratic function of L(t) and a slightly modified single line formula. The original and new formula are compared by applying them to a 6 days long series of real meteorological data.

1. A short reminder of the original results

The expression of the moist enthalpy of air is given as:

$$\mathbf{H = H_a + H_v = (1.007*t - 0.026) + q*(L(t) + 1.84*t)} \qquad \text{[eq.1]}$$

where H_a is the sensible heat and H_v the latent heat plus sensible heat of water vapour, both given in kJ/kg. In this post the small letter t is the temperature in °C; capital T will be used for the temperature expressed in Kelvin ($T = t + 273.15$). The specific humidity q is expressed in kg water per kg dry air.

This formula leads to 2 further problems:

- what is the best expression for L(t), knowing that L does not vary linearly with temperature
- what is the easiest to use, but still acceptable expression for q ?

2. Deriving L(t) from first principles

Following Prof. Fucaloro's suggestion [ref.3], let us start with a fine-grained table of saturated water vapour which can be found on the chemistry webserver-site of Oklahoma State University [ref.1]. For our aim, the relevant temperature range is 0 to 50 °C; the table gives the saturated water vapour pressure e_{asat} for every degree Celsius (pressure is given in mmHg, and must be multiplied by 0.13332211 to obtain kilopascal)

Temperature is converted into Kelvin and inverted to get 1/T; the graph of $\ln(e_{asat})$ versus 1/T is close to linear as shown in fig.1.

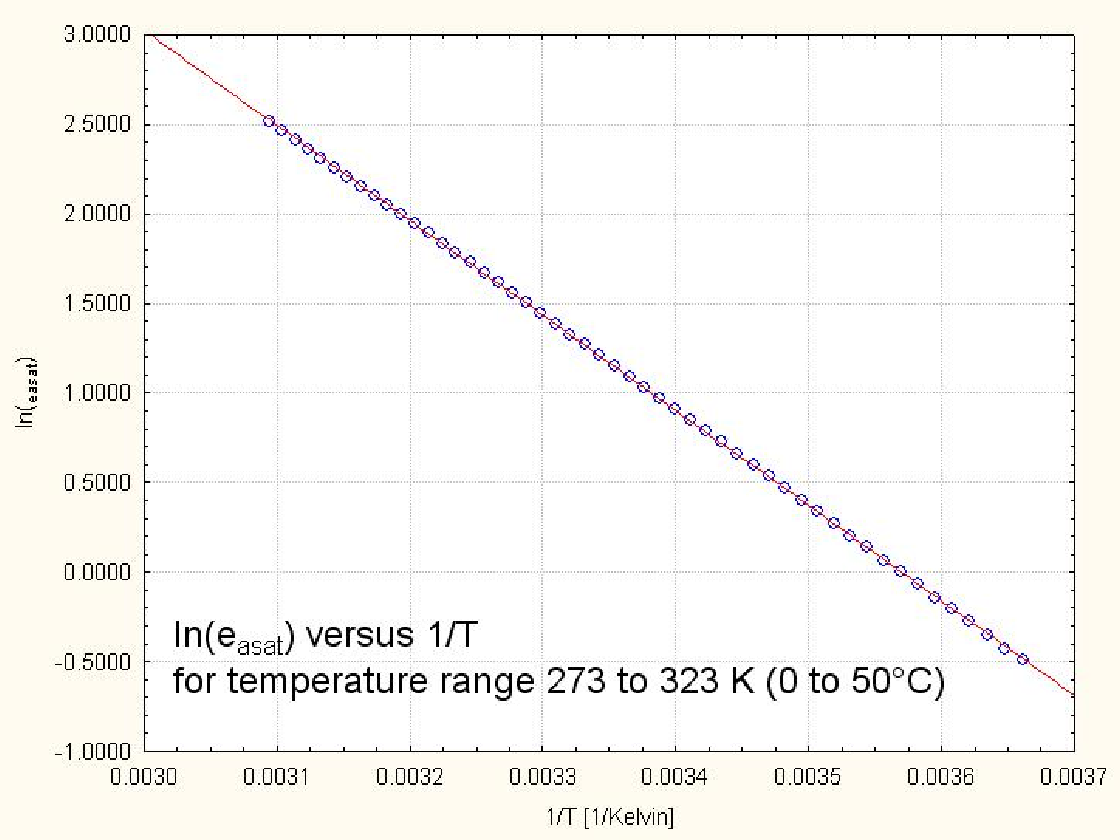

fig.1

Nevertheless we will follow Prof. Fucaloro's post and compute a best cubic fit using the Levenberg-Marquardt algorithm from the Statistica software package. Here the result:

$$\ln(e_{asat}) = 10.9859 + 1065.47 * \frac{1}{T} - 1675 * 10^3 * \frac{1}{T^2} + 14431 * 10^4 * \frac{1}{T^3} \qquad \text{[eq.2]}$$

Remember that e_{asat} is given in kPa, T in Kelvin.
Using eq.3 of Prof. Fucaloro's post the latent heat L(T) can be found as:

$$L(T) = -R * [1065.47 + 2 * 1675 * 10^3 * \frac{1}{T} + 3 * 14431 * 10^4 * \frac{1}{T^2}] \qquad \text{[eq.3]}$$

where R is the universal gas constant 8.3145 and L(T) is given in J per mol. To convert to kJ per kg, L(T) must be multiplied by 1/18. The following figure shows both expressions of latent heat L(t): that of eq.3, with absolute temperature changed to °C, and that given in [ref.2]: L(t) = 2502 – 2.378*t ; the left axis corresponds to the difference between both formulas.

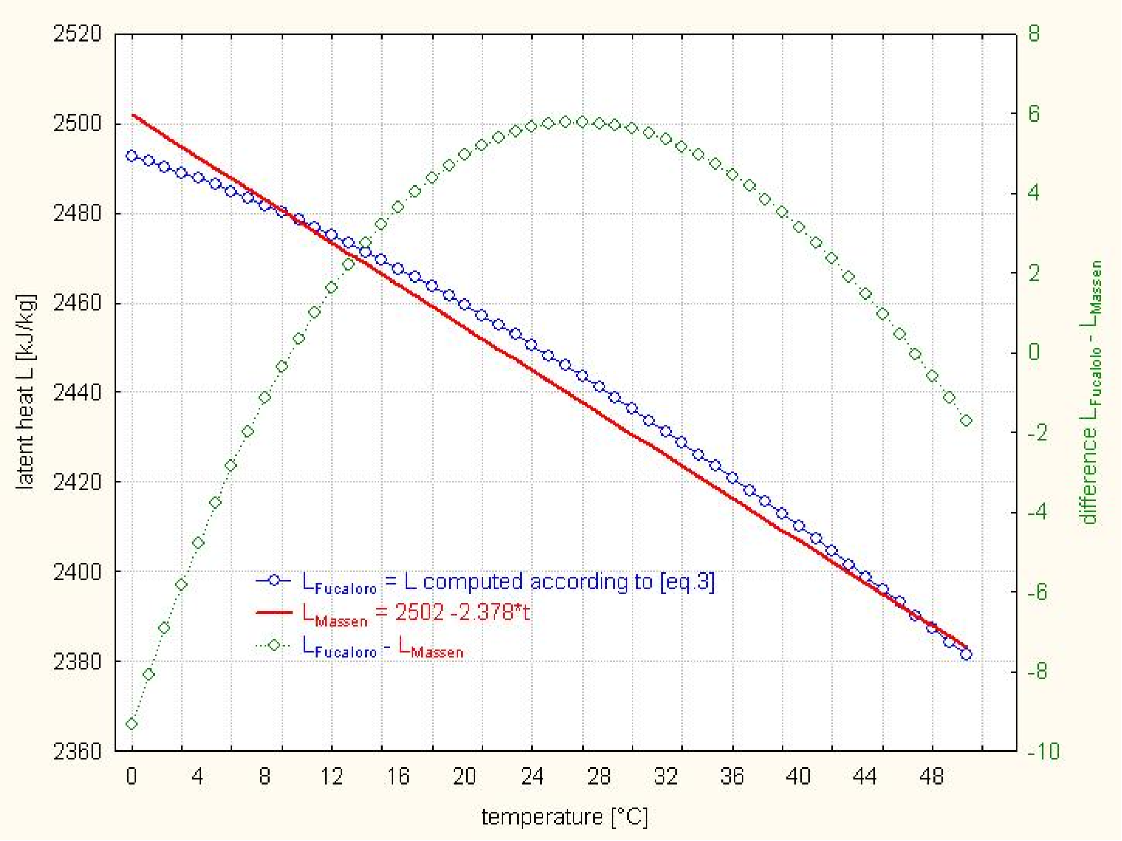

fig.2

The difference is visible, but does not exceed 0.5%. Nevertheless using for L(t) the values obtained by applying eq.3 seems reasonable. A second degree polynomial fit to L(t) gives an excellent result useable in the sought after single line formula:

$$L(t) = 2493.59 - 1.362 * t - 0.01797 * t^2 \qquad \text{[eq.4]}$$

Adapting eq.1 leads to the following expression for the moist enthalpy:

$$\mathbf{H = H_a + H_v = (1.007*t -0.026) + q*(2493.59 + 0.478*t - 0.01797*t^2)} \qquad \text{[eq. 5]}$$

Prof. Fucaloro has added in a recent post [ref.4] that the gas constant R should eventually be replaced by R*z, where z is a compressibility factor. As z seems to be very close to 1, this further complication will be neglected here.

3. Expression to be used for q

The specific humidity q in kg water per kg dry air has been shown to be [ref.5, ref.3, ref.2]:

$$q = \frac{0.622}{\frac{p_a}{e_a} - 0.378}$$

The water vapour pressure e_a = (RH/100)* e_{asat}. To obtain a single line formula, e_{asat} must be expressed as a function of temperature; in [ref.2] the following formula was used:

$$e_{sat} = 10^{\left[\frac{0.7859+0.03477*t}{1+0.00412*t}+2\right]} \qquad \text{[eq.6]}$$

with e_{sat} in Pascal and temperature t in °C. A comparison with the values from the water pressure table shows differences up to 60 Pa (ca. 0.5%). To minimize these differences, the formula in [eq.6] has been fitted to the tabulated values to obtain new numerical parameters:

$$e_{sat} = 10^{\left[\frac{2.193146+0.065876*t}{1.784070+0.007594*t}+1.555267\right]} \qquad \text{[eq.7]}$$

The match is truly excellent, with a nearly ten-fold reduction between the mismatch of the pressure values obtained from eq.6 and those of the table.

4. The revised formula for moist enthalpy

Using the new expression for L(t) and the new parameters of the water vapour pressure formula, the revised formula for moist enthalpy becomes:

$$H = (1.007*t - 0.026) +$$

$$(2493.59 + 0.478*\mathrm{t} - 0.01797*\mathrm{t}^2)*\frac{0.622}{\dfrac{p_a}{\dfrac{RH}{100}*10^{\frac{2.193146+0.065876*t}{1.784070+0.007594*t}+1.555267}} - 0.378}$$

[eq.8]

This expression is certainly valid for temperatures greater than 0 °C and up to 50 °C.

A last question has to be answered: do these further complications matter, or will the differences be minuscule?

To check, the meteorological data from Diekirch for the 6 day period from 2nd to 7th August 2010 have been used. As shown by fig.3, the values for both expressions of H are nearly identical, and are undistinguishable on the plot. The largest difference does not exceede 0.3%:

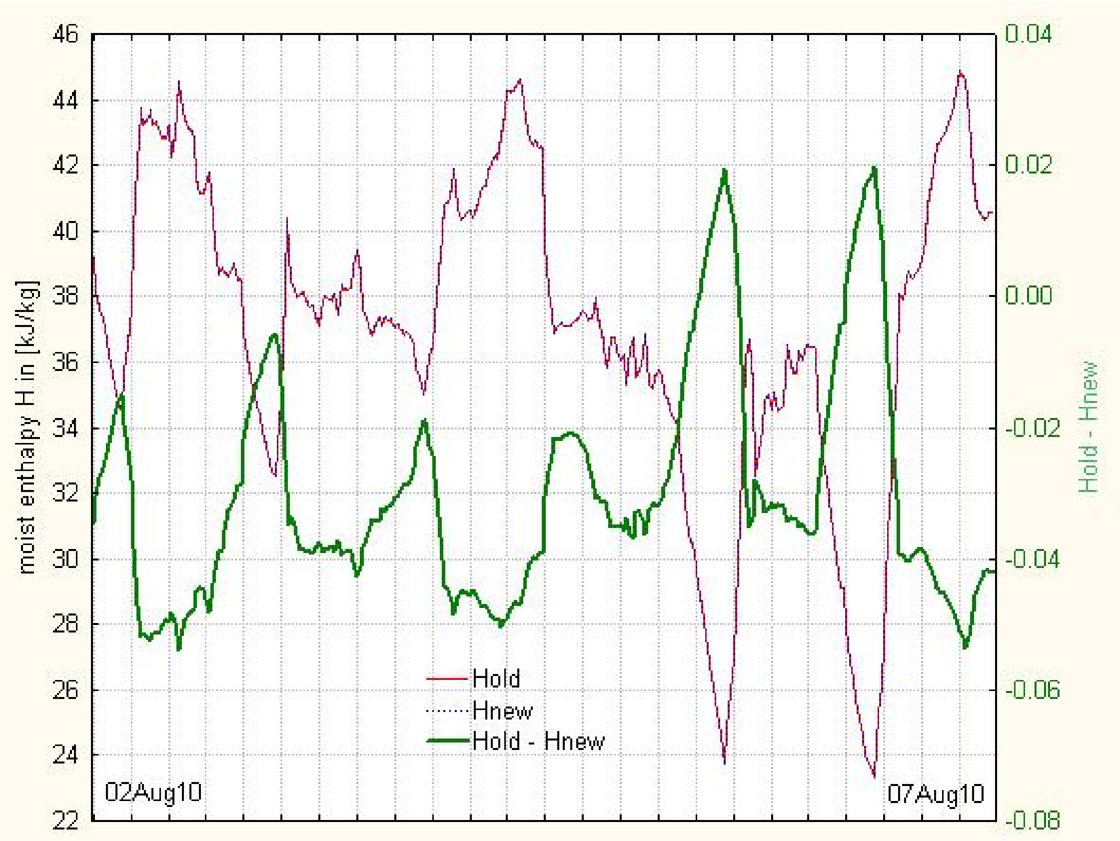

fig. 3

5. Conclusion

The discussion around this important moist enthalpy parameter was very exciting, and I am grateful both to A. Fucaloro and R. Pielke for the very civilized and pleasant exchange of ideas and letters. Probably the last word is not out on this subject, and I wait with great pleasure for the next comments..
I will adjust the formula used at the meteorological station meteoLCD for displaying the moist enthalpy to the new expression; the near-live graphs can be found at http://meteo.lcd.lu/today_01.html

References:

[1] http://intro.chem.okstate.edu/1515sp01/database/vpwater.html

[2] http://pielkeclimatesci.wordpress.com/2010/07/22/guest-post-calculating-moist-enthalpy-from-usual-meteorological-measurements-by-francis-massen

[3] http://pielkeclimatesci.wordpress.com/2010/08/04/comments-on-the-moist-enthalpy-calculation-by-anthony-f-fucaloro/

[4] http://pielkeclimatesci.wordpress.com/2010/09/09/guest-post-by-anthony-f-fucaloro/

[5] PIELKE, Roger, Sr., WOLTER, Klaus: The July 2005 Denver Heat Wave: How unusual was it ?. National Weather Digest, vol.31, no. 1, July 2007
http://pielkeclimatesciencesci.files.wordpress.com/2009/10/r-313.pdf